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Educational Value of Analytically Described Flow Elements

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Abstract

We look upon mathematical solutions to flow equations of motion from the viewpoint of investigating their lasting value in a time when almost every complex boundary value problem in aerodynamics or flow technology can be solved by numerical simulation, employing fast software and powerful computers - and with almost no knowledge about the relevant flow physics needed. Decades ago, when such computing power was not available, researchers created what today is called the theoretical knowledge base which then was used together with experiments for getting started with practical design work in aerodynamics. Here we are particularly interested in preserving the models describing phenomena of inviscid compressible flow, as they not only are valuable for education but also define a rational geometrical basis for accelerated computational configuration design and optimization.

I. Geometry and Fluid Mechanics

During the past century, models for static and dynamic processes in a mechanic and dynamic environment were developed based on learning from nature and on basic solutions to mathematical model equations. For aero- and hydrodynamic vehicles or turbomachinery components, this meant finding and understanding model solutions giving practical hints to the designer of flow boundaries, how to create shapes with high degrees of efficiency, which are usually defined by the parameters clearly related to operation cost, like lift, drag, energy input or output.....

Much of the past successful modelling in describing realistic (compressible, viscous) flows has been used for software development, resulting in commercial tools for Computational Fluid Dynamics (CFD). Academic education of a new generation of engineers has therefore shifted from presenting ad hoc techniques in fluid mechanics to teaching the basics of numerical algorithms solving the Navier-Stokes equations, to mention only the most practical set of model equations covering a wide range of technically relevant flows.

For design purposes, though, numerical analysis may have to be used in a costly large number of iterations, until the finally chosen shapes represent a boundary condition for technically attractive flow quality and finally desirable high degree of efficiency. In this situation, we find it useful to further employ the previously derived model solutions:

Their physical boundaries have certain geometrical property causing the specific modelled flow quality. We therefore like to include the mathematical description of such geometry in the definition of practical design boundary generation, in order to verify the flow model by CFD analysis and beyond this, being able to systematically vary the input parameters for a better understanding of their role in arriving at a flow boundary's optimal technical efficiency.

In the following illustrations and applications we see 'knowledge based geometry' become an important additional component in the design process: Much more intelligent than an arbitrary commercial Computer Aided Design (CAD) software, it accelerates the design optimization process by leading the designer to the most important shape variation parameters.

Besides this practical aspect and in focus here, the educational value for presenting the evolution of knowledge based design tools to the new generation of engineering students seems obvious:

A limited, but well organized package of illustrative basic mathematical models along with their geometrical appearance, and finally a rich collection of shape definition functions extending theory to practical preprocessing of commercial software should be welcome to graduating creative students.

This goal in mind, the author tried to extract suitable practical models from his earlier theoretical work in order to show the continous usefulness of theory essentials, and to suitably highlight them within the research topics of transonic fluid dynamics [1] and high speed air transport technology [2].

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II. Compressible flow and its analytical models

With various options to choose from commercial CFD software analyzing flows with realistic boundary conditions it seems anachronistic to still mention one of the older flow modelling concepts like ‘Potential Flow’, but for design purposes this modelling concept continues to give the fastest practical tools when aerodynamically favorable configurations are sought - which is usually the design goal. The reason for this is the absence of viscosity and energy sinks in potential flow: This also marks the ideal (though unrealistic) design goal of vanishing viscous and shock losses. This fact was used by Takanashi [3] when employing the potential flow concept for the *inverse design step* of prescribing target performance, but the full Navier Stokes CFD for the *direct analysis step* in an iterative procedure.

A restriction to potential flow for selection of models with both practical and educational value seems therefore appropriate and it is worth to preserve the classical, most descriptive, special solutions of potential flow examples for being coupled with modern design and optimization concepts. Such special solutions have been found for compressible two-dimensional (2D) flows as they are representing plane flows past wing sections or axisymmetric flows in or past circular cross sections. Many fully three-dimensional (3D) configurations in transonic and supersonic potential flow can be constructed from such 2D models. The knowledge base for realistic (3D) applications has therefore been developed mainly using 2D potential flows, which is a good reason to continue using this material in coursework for aerospace studies.

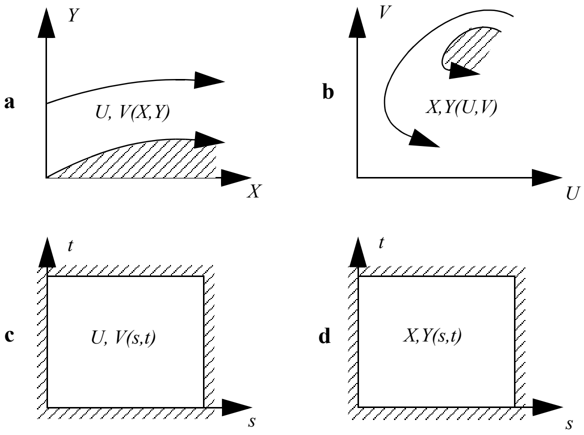
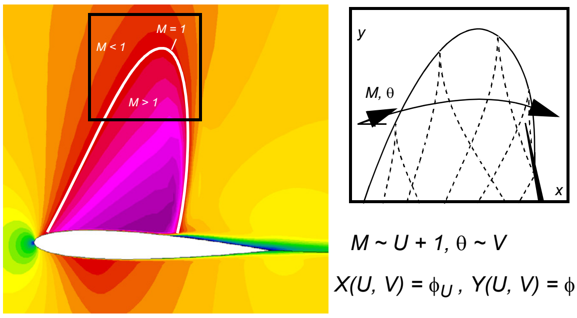


Figure 1. Direct, inverse and parametric representation

Options to describe a flow model and its boundary: (a) In physical space, (b) in a space of state, (c + d) in a parameter space



$$M \sim U + 1,\ \theta \sim V$$

$$X(U, V) = \phi_U\ ,\ Y(U, V) = \phi_V$$

$$\phi = C_1UV + C_2V(V^2 + U^3) + C_3U(V^2 + U^3/6) + ..$$

Figure 2.Flow models embedded in realistic aerodynamics.

Analytical solutions illustrate compressible flow phenomena: Details of transonic airfoil flow with recompression shock and local description of a shock formation in the flow field

Mapping the problems to become linear

It is a challenge to stress the value of mapping a complex scientific problem - as described by non-linaer equations - from physical space to a working space, if the transformed problem in this new space becomes simpler, like being represented now by a set of linear differential equations. In the case of 2D compressible flow with small perturbations, the effort of studying the problem in a new working plane is rewarded not only by linearity of the equations, but also by options to control the boundary conditions. The evolution of such mapping concept is sketched in Figure 1:

A flow element in 2D space (X, Y) may be described by its velocity distribution U,V(X,Y), see Fig.1a. For compressible potential flow, all other variables of state (pressure, density, temperature) are dependent of velocity via the energy equation, the components U, V are gradient of a single variable, the potential. The structure of the basic differential equations allow for transformation of the variables which results in a similar representation:

The physical coordinates may be found as a function of variables of state: X,Y(U,V), see Fig.1b. This is called a hodograph formulation. It has the advantage of the differential equations becoming linear, for another (so-called Legendre-) potential, the gradients of which are now the physical coordinates X, Y. The advantage of linear equations is obtained at the cost of a non-linear boundary value problem, which means that physical boundaries are usually appearing in a non-illustrative shape. Hodograph methods have, therefore, been not popular in the past with engineers because of the abstract character of related illustrations. Nowadays with graphic software, the inversion of the results to show U,V(X,Y) and comparison with numerical or experimental analysis is of course quite practical (Figure 2).

Indeed a compromise keeping the advantage of linear equations and also more control of boundary conditions is a normalization resulting from introducing a ‘neutral’ parameter plane, represented by some new coordinates (s,t): Both the variables of state and the physical coordinates are now functions of the independent parameters, X,Y(s,t) and U,V(s,t). For 2D (plane) compressible flow, we solve 2 sets of linear differential equations. For axisymmetric transonic flow a non-linear coupling between the equations remains, but non-linearity is quite weak, so that technically interesting boundary value problems are solved with about the same computational effort as required for the linear equations. The system of differential equations is mathematically known as Beltrami equations, which are of either elliptic or hyperbolic type:

Elliptic Beltrami equations are a generalization of the Cauchy Riemann mapping equations, they represent the subsonic flow regime.

Hyperbolic Beltrami equations are generalized wave equations, valid for the supersonic flow regime and a basis for linear numerical methods of characteristics.

For a mixed subsonic-supersonic flow, called transonic flow, the independent parameters (s,t) allow a clear distinction of the problems in neighboring domains, with contact along a ‘sonic locus’. The structure of the common values X, Y, U, V along this mapped isotach where Mach = 1, determines the physical structure of the flow. A unified representation of subsonic, transsonic, supersonic, plane or axisymmetric flow in this parametric form is given by Sobieczky [4], with explanations of solution parameters for some physically relevant flow models.

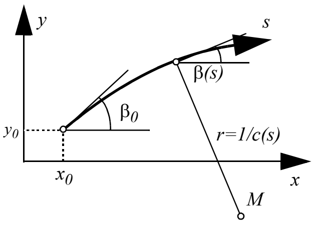
Using an analogy to construct flows

The structure of Beltrami equations for X,Y(s,t) or U,V(s,t) modelling subsonic compressible flow is equivalent to plane electric potential distributions within a network of variable conductivity (Rheoelectric Analogy): A plane, shallow tank with an electrolytic fluid (water) was a classical, inexpensive ‘Poisson Solver’ with immense educational value to understand potential problems and mapping procedures in the past century, until the first resistor networks were built and used as early analog computers, see Malavard [5]. The author of this paper has enjoyed the verification of exact solutions to the elliptic Beltrami equations by such network setups; we have called the above mentioned working plane (s,t) a ’rheoelectric hodograph’ or „Rheograph“ plane. But soon enough we found that numerical solvers in the meantime allowed for a time-economical replacement of such analog methods in mapped working planes. The educational value of playing with the analog, though, especially for transonic applications with its mixed boundary value problems was extremely useful for the later, purely numerical approaches to design of aerodynamic shapes by choosing efficient design parameters. An illustrative paper of the work phase when transition from analog to numerical approach was performed, has been presented by Sobieczky [6].

From gasdynamic phenomena to surface curvature and slope control

The special steps of constructing transonic flow examples in the Rheograph plane soon were paralleled by the equivalent numerical approach in physical space, where they could be applied not only to 2D airfoil flows but as well to realistic 3D aircraft and turbomachinery components. First procedures of this kind were called „Fictitious Gas“ design, because the translation of the mathematical concept to solve boundary value problems in a Rheograph plane into physical space meant a temporary introduction of a gas with artificial properties, just to support an intermediate design step before physics finally became ’real’ again when the design was completed. In later years, when flow control technology started to be important for improved aerodynamics we interpreted the fictitious gas as a realistic one with a controlled heat addition and subtraction, some kind of ’gasdynamic flow control’ .

The most practical and quite simple knowledge harvested from these early and illustrative techniques was to learn about qualitative and quantitative surface geometry changes required to modify a given configuration from arbitrary or unfavorable aerodynamic performance to become a good candidate for successful high speed wing design: In *transonic aerodynamics*, most effective changes in flow quality are obtained by delicate *local curvature* changes. Input data controlling surface geometry with given curvature prescription results in useful design parameters: Local curvature control along a contour, or given along an axis, c(x), is obtained by numerical integration of the Natural Equation with osculating circles or parabolas, Fig. 3. Curvature distributions from flow models obtained by the above mentioned inverse techniques guide us to use generating functions with suitable free parameters to arrive at families of contours whith controlled aerodynamic performance. Implementation of exact solutions, but also their suitable approximation in our design tools is part of the *aerodynamic knowledge base* [Ref. 2, chapter 7] derived from the earlier theoretical approaches.



$$\beta(s) - \beta_0 = \int_0 c(s)ds$$

Figure 3.Curvature and slope control.
Using the Natural Equation to create a contur from given curvature c(s)

In *supersonic aerodynamics*, the use of analytic solutions is even easier: Flow models with oblique 2D or 3D oblique shock waves provide representative streamlines which serve as generatrices for quite arbitrary 3D flow boundaries being compatible with the shock wave. It is predominantly the *local slope* of the boundary which determines supersonic velocity and pressure distribution.

III. Geometry based on fluid dynamics

A refined knowledge of the most efficient parameters to influence flow qualitity in the respective speed regimes has been gained from the briefly mentioned analytical theories. For some decades these theories have been used for practical design work, but with faster computers and the availability of numerical flow analysis, this knowledge base needs to be made useful in the input data definition of common analysis and optimization software.
Results from 2D flow modelling therefore were used to suitably parameterize spanwise wing sections, i. e. airfoil shapes with favorable ratio lift/drag, and axial cross sections of minimum drag for given volume.

Airfoils

Early systematic research to study the performance of airfoils in wind tunnels led to catalogs for airfoil families (NACA, NASA) and their drag polars in various operating conditions. Today, with reliable CFD analysis to provide airfoil performance data, it is rather necessary to create airfoil coordinates and immediately optimize them for the special mission. In order to have a multiplicity of possible airfoil shapes to choose from for optimization, suitably parametretized shape functions have been provided as “Parametric Sections” - PARSEC XX - with XX defining the number of free parameters. With a first set of 11 parameters already most of the important curvature details suggested by the analytical model solutions may be influenced, Sobieczky [7]. Additional parameters provide models for local modifications as they would result from concepts requiring special technically feasible devices, like bumps near the sonic expansion area, Sobieczky et al [8], leading and trailing edge modifications.

Cross sections and given shock wave

An important topic of classical supersonic flow theory was finding body shapes of given caliber or given volume to result in minimum drag when flying with supersonic Mach numbers. The limits of wing theory, if wing span becomes very small, overlaps the theories for slender bodies in high speed flow, axial distribution of the body volume becomes more important than the detailed shape of the cross section. Various basic theories are contributing to define supersonic wing and body coss section geometries along the axis. With recompression shocks contributing substantially to drag production, it is desirable to use the inverse concept of defining shock wave geometries as an input for finding the compatible body causing it in supersonic free stream flow. There are analytical models for flow past a wedge or a cone, each causing an oblique shock. These are classical material to educate students and ideally suited to explain their use for describing special flow elements with shock by selecting only parts of these models:
Three-dimensional ramp flows are created with shock waves of a given strength. We have generalized these design principles by recognizing the local character of interacting cone surface and oblique shock in a cross section plane, [Ref 2, chapter 8]. Euler equations method of characteristics and numerical marching is used to further generalize this inverse design principle, Qian and Sobieczky [9], for curved shocks which give additional freedom in shape generation of realistic convex or facetted forebodies applicable to diffuser and missile forebody design.

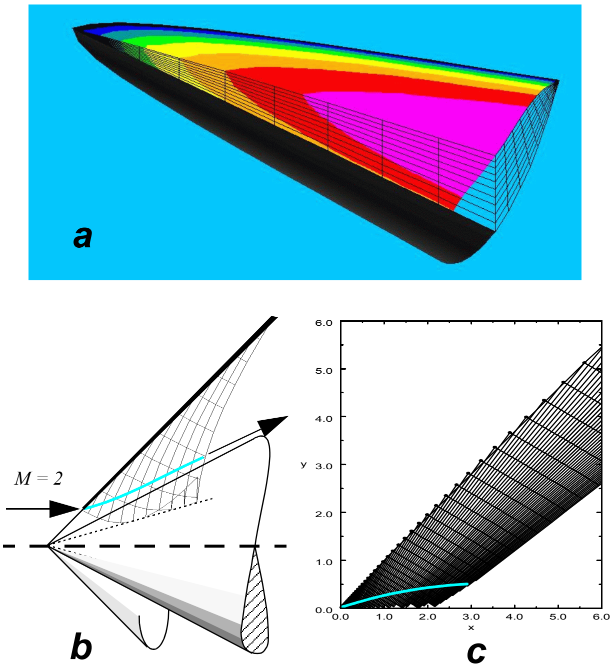


Figure 4.Shock compatible with design geometry
*Selection of supersonic cone flow with shock segment defines stream surface with given pressure distribution (**a**). Cone flow verification by inverse marching from given shock cone, (**b**). Given curved shock wave results in convex contour for 2D or axisymmetric forebodies, (**c**)*

Geometry Preprocessor

The above mentioned functions to create 2D boundaries, in suitable planes within 3D space, have been coded as the intelligent part of what otherwise would be just another CAD surface generator. It turned out that an availability of basic functions with not more than 4 input parameters is sufficient to create any interval with controlled curvature and slope between supports a complex curve in 2D space. These functions serve either as suggested by approximating analytic flow model solutions, or to define scaled configuration data defined by technical three-view drawings [Ref. 2, chapter 9]. Software tools have therefore been written to generate aerodynamic configuration surface coordinates, including whole families of shapes morphing from one extreme to another, in order to choose from the data for numerical optimization, for adjusting adaptive devices or for modelling unsteady boundary conditions as they occur in nature.

Most practical design work is done with commercial CAD programs, it is therefore necessary to link our phenomena based geometry tools to such common software. This is done by output formats which are understood by commercial software. Iges formatted or - accepted by only few CAD programs - importing just dense surface grid data are input for these programs which are required to re-define the shape as close as possible to the original but in order to exploit their commonality the engineer will need to continue with this software.

Very fast data generation without many of the problems ('water-tight surfaces', surface wiggles,...) typically associated with CAD work results from this introduction of a smart precursing step prior to geometry data production.

Figure 5 shows the usual initial design and optimization loop, extended in the front end by adding elements of the knowledge base and a dedicated parameterized surface generator to define what further can be processed by commercial CAD and analysis software.

High speed aircraft wing design toward innovative configurations and the definition of hypersonic forebodies from basic oblique shock solution to complete aerospace plane configurations are the following topics, to illustrate theoretical and numerical studies prior to regular project work.

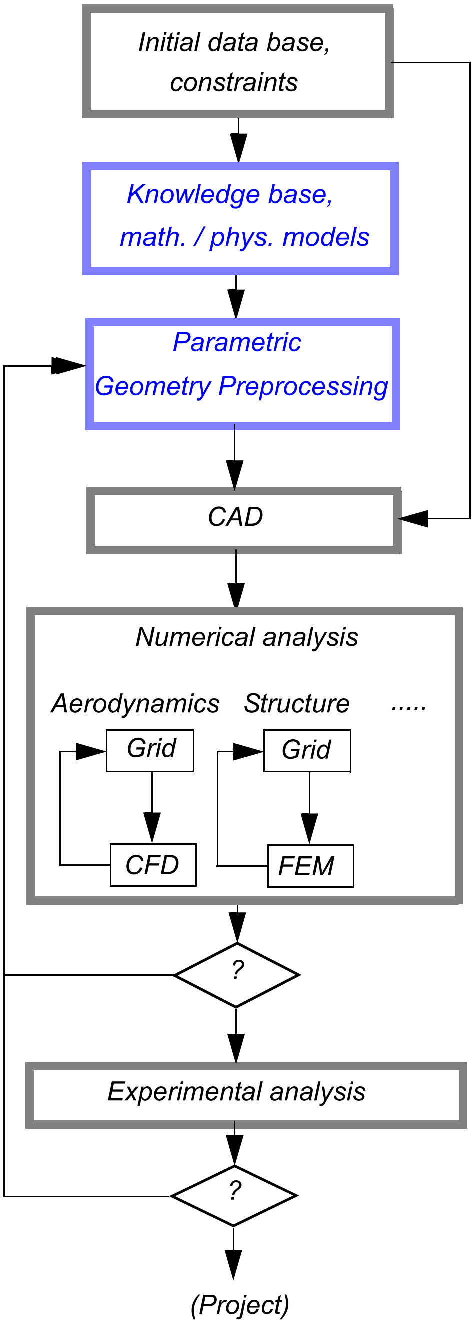


Figure 5.Design procedures.
Parametric geometry preprocessing provides a smart CAD input for subsequent multidisciplinary optimization

IV. Configurations with suitable design parameters

Many theoretical details of design concepts get lost if they are not contributing directly to improving efficiency of practical case studies. With modern communication and documentation methods it is easy to keep these things alive, so we may create educating documentations to serve as teaching material available also to the design engineer having to solve practical tasks in the real professional world, without much effort usually associated to the search in libraries and old file systems. For applied aerodynamics, the above sketched preprocessing is intended to represent such concentrated knowledge. Here and for illustrating transonics, we mention a good acceptance of our PARSEC airfoils by other authors, who have used them in their optimization strategies for aircraft wings and benefitted from their implicit curvature management in the used functions.

Aircraft Wings
Klein and Sobieczky [10] use various airfoil definition functions and compare the resulting pareto front of shapes with optimum aerodynamic performance. It can be seen that functions with more relationship to the design knowledge base (as represented for instance by curvature control) result in superior performance. Oyama [11] compares well-known airfoil series with PARSEC 11 and finds them superior, both regarding aerodynamic efficiency and economic convergence rate in a costly optimization based on evolutionary algorithms. Holst and Pulliam [12] similarly observe economic advantage when using these airfoils for genetic algorithm (GA) based wing optimization which is found useful also for multidisciplinary (aerodynamic - structural) optimization.

Configuration Integration
Parametric definition for the preprocessor usually provides airfoil-type sections for 3D wings and cross sections for 3D fuselages. But alternatively, small aspect ratio wings may be created as body-type surfaces and flat bodies suggest a description using longitudinal sections like airfoils. This flexibility and adherence to cartesian coordinates definition is found useful to adapt surface metrics to various practical purposes dictated by the subsequent use of commercial analysis codes. Cartesian description makes it relatively easy to project wing sections near the root toward the body in order to ensure a smooth junction without corners. Intersection of the components is found routinely by CAD, but smooth tangential contact lines are difficult for CAD to find. We prefer, therefore, to provide wing plus fuselage surface data to CAD input as one surface, by changing the body surface metric as a continuation of the wing sections description from the root toward the center plane of symmetry. Flow quality at each component intersection (e. g. pylons and wing) should be subject to parametric variation and analysis, but the wing root seems the most important integration task. Figure 6 illustrates visualization of a generic transport aircraft, showing the main components for parametric aerodynamic variation study. Further options of the preprocessor include the composition of the wing by multicomponent elements to provide data for simulation of high lift configurations.

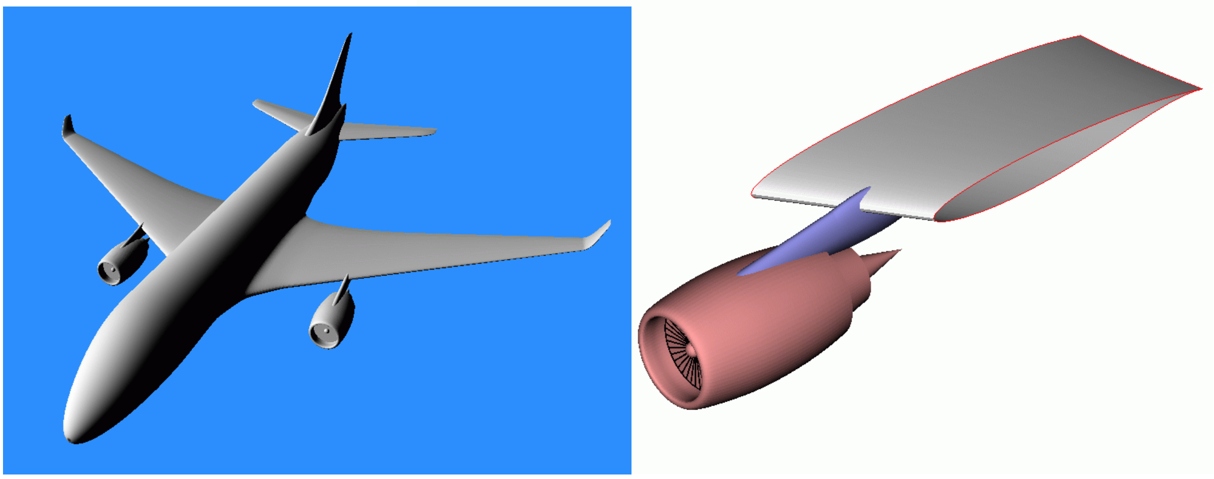


Figure 6.Generic transport aircraft baseline geometry .
Winglets are included in wing spanwise surface definition. Wing and fuselage may be generated as one surface ensuring effective fillet quality control.
Pylon is generated with the wing tool, for nacelle bypass cowl the body tool is used for axisymmetric, or the wing tool for non-axisymmetric (ring-wing) cross sections.

Input for multidisciplinary analysis and optimization
The CAD step following the parametric surface definition provides input for unstructured grid generation which is required for subsequent analysis with a CFD code solving the Navier-Stokes equations. For structural analysis, the same surface grid will be used to generate grids for the interior of the solid boundary: Both grids together are reqired for aeroelastic design and optimization. Truly formidable tasks for multidisciplinary work exist in turbomachinery, where rotor/stator blade aerodynamics have to be balanced with heat loads, centrifugal and aerodynamic forces on the blades, heat transfer via the blade to a coolant fluid in suitably shaped channels within the blades. Dennis et al [13] have used the preprocessor functions for the first steps into such work.

Innovative configurations

The option to easily generate arbitrary surfaces with a potential to be applied in new concepts and projects of the aerospace industry should also encourage creative people to use generic, preliminary and innovative configuration geometries for the development and the demonstration of their respective ideas. For aerodynamics, we have done so in a very modest scale in the past decades, but teams working in applied aerospace R&D with more practical experience have picked up some of the models and concepts to develop, refine and use their powerful design tools for the high speed flow regime, see also [1], [2].

One of the most challenging aircraft concepts for supersonic transport is the Oblique Flying Wing (OFW), a wing-only configuration flying with a yaw angle adjusted to the flight Mach number. We have optimized a single wing in supersonic flow using classical analytics plus CFD to calibrate the wing geometry parameters in order to minimize shock wave drag, Li et al [14]. Quite recently, the iges-formatted geometry data resulting from this optimization have been used by Heyl [15] for a design study to propose solutions for interior aircraft design and operation on airports. This is an example of fruitful data creation starting from classical analytics to numerical analysis and dissemination for further use. Figure 7 illustrates the various steps of this work:

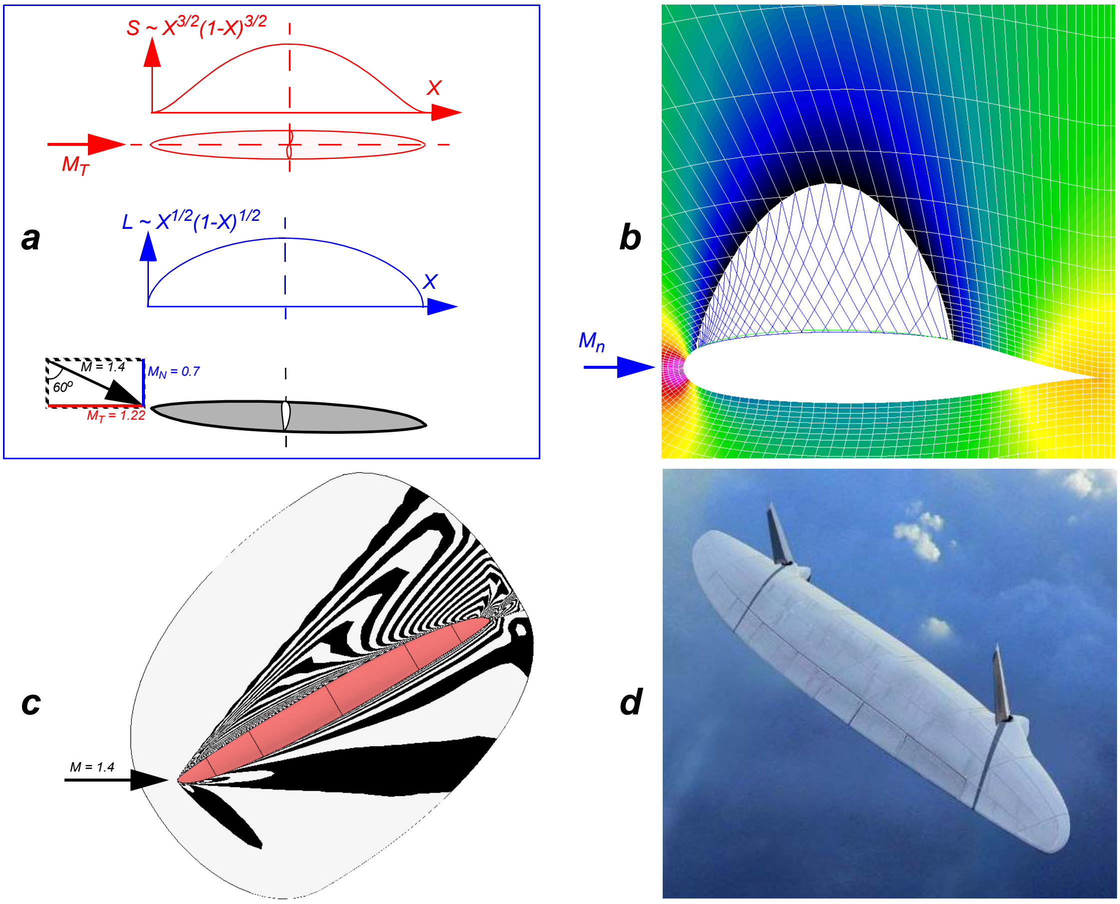


Figure 7.Oblique Flying Wing Design

*Basic theoretical aerodynamic models include equivalent body of revolution for supersonic ($M_T < 1$) minimum drag (Sears-Haack body) and elliptical wing lift loading, (**a**). Spanwise variation of transonic ($M_N < 1$) design of thick wing sections, (**b**). CFD analysis and shape variation for optimum aerodynamic efficiency, (**c**). Design studies for complete aircraft, interior design and operation at airports, (**d**).*

Hypersonic applications

Hypersonic flow is defined by the supersonic domain with flight Mach numbers above about 4, where thermal effects seriously compromise the quantitative validity of gasdynamic models obtained from potential or Euler equations. Nevertheless we use supersonic flow elements with higher Mach numbers and oblique shocks for starting configurations in a design process, to control shock strength, and hence wave drag, by surface contour slopes. The waverider design principle exploits the close relationship between shock- and surface-geometry:

Inverse techniques have been developed and made applicable in design software. Input is a combination of 3D surface geometry characteristics and 3D shock shapes. Center [16] developed interactive software for aerospace vehicle optimization. The goal is to define forebody surfaces serving as diffusers providing parallel compressed flow at the inlet plane, for ramjet or scramjet propulsion. Eggers and Novelli [17] have carried out a numerical and experimental design study ‘JAPHAR’ with an optimized forebody based on results of this software:

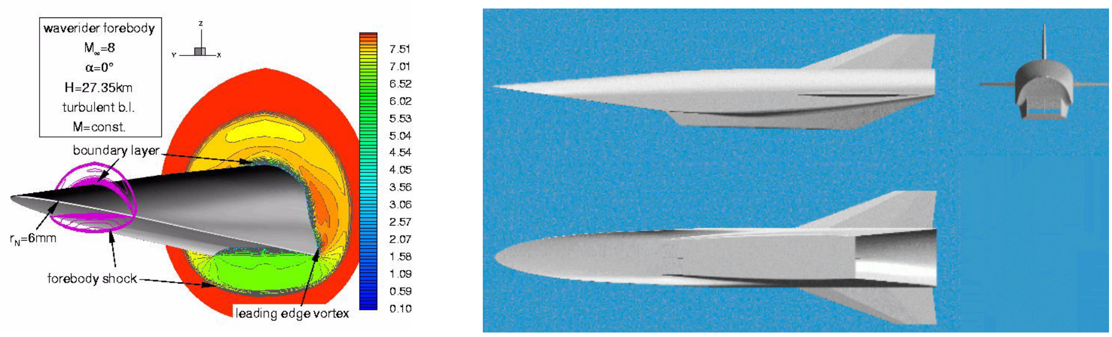


Figure 8. Project JAPHAR
CFD hypersonic flow analysis of a waverider forebody used for design optimization of an experimental Mach 8 aerospace vehicle.

V. Concluding remark

Purpose of this contribution is to focus on the value of keeping alive a connection between theoretical flow modelling and modern aerospace design approaches. This value consists, (1), of a better understanding of the background for present numerical simulation and, (2), a rational basis for modern configuration design methodology which is useful in economical optimization strategies. Both aspects should provide material to make aerospace education more interesting especially to creative students. A result of combining the basics with the requirements of efficient design engineering is the strong role of analytical geometry tools which are found useful as a preprocessor of CAD input data.

References

[1] Sobieczky, H., (Ed.): Symposium Transsonicum IV, Proceedings of the IUTAM Symposium, Kluwer Academic Publishers (2003)

[2] Sobieczky, H., (Ed.): New Design Concepts for High Speed Air Transport. CISM Courses and Lectures Vol. 366. Wien, New York: Springer (1997)

[3] Takanashi, S.: An Iterative Procedure for Three-Dimensional Transonic Wing Design by the Integral Equation Method. AIAA 84-2155, (1984)

[4] Sobieczky, H.:Transonic Singularities. Comp. Fluid Dynamics J. Vol 12 No 2 (2003), pp. 415-419

[5] Malavard, L. C.: The Use of Rheoelectrical Analogies in Aerodynamics. AGARDograph 18 (1956)

[6] Sobieczky, H.: Related Analytical, Analog and Numerical Methods in Transonic Airfoil Design. AIAA 79-1556 (1979)

[7] Sobieczky, H.: Parametric Airfoils and Wings. In: K. Fuji and G. S. Dulikravich (Eds.): Notes on Numerical Fluid Mechanics, Vol. 68, Wiesbaden: Vieweg (1998);

[8] Sobieczky, H., Geissler, W., Hannemann, M.: Expansion Shoulder Bump for Viscous/Wave Drag Control. In: G. E. A. Meier (Ed.) Proc. IUTAM ‘FLOWCON’ Symposium on Mechanics of Passive and Active Flow Control. Kluwer Academic Publishers (1998)

[9] Qian, Y., Sobieczky, H., Eggers, Th.: Waverider Design with Parametric Flow Quality Control by Inverse Method of Characteristics. In: M. Tanaka, G.S. Dulikravich, (Eds.), Inverse Problems in Engineering Mechanics, Proc. Int. Symp. on Inverse Problems in Engineering Mechanics (ISIP2000), Nagano, Japan (2000)

[10] Klein, M., Sobieczky, H.: Sensitivity of Aerodynamic Optimization to Parameterized Target Functions. In: M. Tanaka, G.S. Dulikravich, (Eds.), Inverse Problems in Engineering Mechanics, Proc. Int. Symp. on Inverse Problems in Engineering Mechanics (ISIP2001), Nagano, Japan (2001)

[11] Oyama, A.: Wing Design Using Evolutionary Algorithm. Ph.D. Thesis, Tohoku University, Sendai (2000)

[12] Holst, T. L., Pulliam, Th. H.: Transonic Wing Shape Optimization Using a Genetic Algorithm. In: Sobieczky, H., (Ed.): Symposium Transsonicum IV, Proceedings of the IUTAM Symposium, Kluwer Academic Publishers, pp. 245-252 (2003)

[13] Dennis, B. H., Egorov, I. N., Sobieczky, H., Dulikravich, G.S., Yoshimura, S.: Parallel Thermoelasticity Optimization of 3-D Serpentine Cooling Passages in Turbine Blades. ASME GT2003-38180 (2003)

[14] Li, P., Sobieczky, H., Seebass, A. R.: Manual Aerodynamic Optimization of an Oblique Wing Supersonic Transport, J. Aircraft Vol.36, No.6, pp 907-913, (1999)

[15] Heyl, F.: "Aeolus - the Oblique Flying Wing", 6th Aircraft Interiors Expo 2005, Apr. 5 - 7, (2005)

[16] Center, K. B.: An interactive Approach to the Design and Optimization of Practical Hypersonic Waveriders. Ph.D. Thesis, University of Colorado, (1993)

[17] Eggers, Th., Novelli, Ph.: Design Studies for a Mach 8 Dual Mode Ramjet Flight Test Vehicle. AIAA 99-4877 (1999)